Log Normal Distribution

A continuous distribution in which the logarithm of a variable has a normal distribution. It is a general case of Gibrat’s distribution, to which the log normal distribution reduces with and . A log normal distribution results if the variable is the product of a large number of independent, identically-distributed variables in the same way that a normal distribution results if the variable is the sum of a large number of independent, identically-distributed variables.

The probability density and cumulative distribution functions for the log normal distribution are

\[ P(x) = \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{(\ln x - \mu)^2}{2\sigma^2}} \]  
\[ F(x) = \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{\ln x - \mu}{\sigma \sqrt{2}} \right) \right] \]  

where \( \text{erf} \) is the erf function.

It is implemented in Mathematica as \( \text{LogNormalDistribution}[\mu, \sigma] \).

This distribution is normalized, since letting \( y = \ln x \) gives \( dy = dx/x \) and \( x = e^y \), so

\[ \int_{0}^{\infty} P(x) \, dx = \frac{1}{\sqrt{2\pi} \sigma} \int_{-\infty}^{\infty} e^{-\frac{y^2}{2\sigma^2}} \, dy = 1. \]  

The raw moments are

\[ \mu'_1 = e^{\mu + \frac{\sigma^2}{2}} \]  
\[ \mu'_2 = e^{\mu + \sigma^2} \]  
\[ \mu'_3 = e^{\mu + 3\sigma^2} \]  
\[ \mu'_4 = e^{\mu + 4\sigma^2} \]  

and the central moments are

\[ \mu_2 = e^{2\mu + \sigma^2} \left( e^{\sigma^2} - 1 \right) \]  
\[ \mu_3 = e^{3\mu + 3\sigma^2} \left( e^{\sigma^2} - 1 \right)^2 \frac{(e^{\sigma^2} + 2)}{3} \]  
\[ \mu_4 = e^{4\mu + 6\sigma^2} \left( e^{2\sigma^2} - 1 \right)^2 \left( e^{3\sigma^2} + 2e^{2\sigma^2} + 3e^{\sigma^2} - 3 \right). \]  

Therefore, the mean, variance, skewness, and kurtosis are given by

\[ \mu = e^{\mu + \frac{\sigma^2}{2}} \]  
\[ \sigma^2 = e^{2\mu + \sigma^2} \left( e^{\sigma^2} - 1 \right) \]  
\[ \gamma_1 = \frac{\sqrt{e^{\sigma^2} - 1}}{\left(1 + e^{\sigma^2}\right)^{\frac{3}{2}}} \]  
\[ \gamma_2 = e^{3\sigma^2} + 2e^{2\sigma^2} + 3e^{\sigma^2} - 6. \]  

These can be found by direct integration.

http://mathworld.wolfram.com/LogNormalDistribution.html
and similarly for $\sigma^2$.

Examples of variates which have approximately log normal distributions include the size of silver particles in a photographic emulsion, the survival time of bacteria in disinfectants, the weight and blood pressure of humans, and the number of words written in sentences by George Bernard Shaw.

SEE ALSO: Log-Series Distribution, Logarithmic Distribution, Weibull Distribution

REFERENCES:


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