Troposphere–Stratosphere Interactions in a One-Dimensional Model of Jovian Photochemistry

BRIDGET LANDRY,* MARK ALLEN,*,† AND YUK L. YUNG*

*Division of Geological and Planetary Sciences, California Institute of Technology, Pasadena, California 91125, and
†Earth and Space Sciences Division, Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California 91109

Received April 9, 1990; revised September 4, 1990

A simple one-dimensional model of the Jovian atmosphere including the coupling between a rapidly mixed troposphere and a stagnant stratosphere is presented. We treat analytically the case of a chemically unreactive species flowing downward through the stratosphere and troposphere with a constant flux. The calculated concentration profile has a maximum value approximately one atmospheric scale height above the tropopause. The corresponding mixing ratio rapidly decreases in the lower stratosphere. The contrast between the peak stratosphere and tropopause concentrations reflects the variation between the lower stratosphere and upper troposphere eddy diffusion coefficients. Numerical simulations of unreactive CO and C2H6, considering high-altitude photochemical sources (yielding downward fluxes to the troposphere) and deep troposphere thermochemical sources, demonstrate that upper troposphere abundances may have a large photochemical contribution. Recent observations of CO near 5 bar can be reproduced by a model using any one of three different tropospheric eddy diffusion profiles: a constant value of ~10^8 cm^2 sec^{-1}, a constant value of ~10^5 cm^2 sec^{-1}, or a two-layer model with a rapidly mixed (10^8 cm^2 sec^{-1}) layer below 20 bar and a slower mixing layer (10^4 cm^2 sec^{-1}) between 100 mbar and 20 bar. In the latter two scenarios, the photochemical source is an important and/or dominant source of upper tropospheric CO. However, the upper tropospheric C2H6 abundances are distinctly different among the three cases. These calculations suggest objectives for higher spectral resolution observations, critical vertical scales for planning experiments on future missions to Jupiter, and predictions that can be tested with the Galileo probe mass spectrometer. © 1991 Academic Press, Inc.

INTRODUCTION

It is convenient in modeling the chemistry of the atmospheres of the Jovian planets to divide the chemistry into two distinct regimes. The upper atmosphere above 100 mbar is normally driven by photochemical processes initiated by the absorption of ultraviolet photons (Strobel 1983). The chemical composition in the deep atmosphere at pressures greater than a kilobar is dominated by thermal equilibrium chemistry (Lewis and Prinn 1984). We therefore expect the troposphere to be the region which is influenced by the composition of the atmosphere above and below it.

Previously published one-dimensional (1D) photochemical models of outer solar system chemistry typically have the lower boundary placed at the tropopause (for example, Strobel 1969, 1974; Atreya 1982; Yung et al. 1984). Consequently, there are no calculated tropospheric profiles for species formed photochemically in the stratospheres. Observers therefore model their measurements of stratospheric infrared emission with species profiles that include zero abundances for the species in the troposphere (see, for example, Noll et al. 1986). Profiles with constant mixing ratios in the stratosphere are usually adopted by the observers since the photochemical models, with the lower boundary at the tropopause, typically produce profiles of this form for the long-lived species in the lower stratosphere.

In this paper, we shall present a simple 1D photochemical model including both the stratosphere and the upper troposphere (with the lower boundary set deep in the troposphere) to remedy the aforementioned deficiencies in published models to date. The stratospheric profiles of long-lived species will differ from what has been published and the tropospheric abundances will be quite significant. Both predictions of the calculations may be tested in the next few years by new ground-based and spacecraft experiments.

STRATOSPHERIC PROFILES IN A TROPOSPHERE–STRATOSPHERE MODEL

Since 1D models of outer solar system stratospheres typically adopt an eddy diffusion coefficient $K(z)$ which

---

1 Contribution 4716 from the Division of Geological and Planetary Sciences, California Institute of Technology, Pasadena.
FIG. 1. (a) Discontinuous eddy diffusion coefficient utilized in analytic calculations discussed in text. Above the level of discontinuity, the eddy diffusion coefficient scale height is twice the atmospheric scale height. (b) Corresponding analytic model results (solid lines) for number density \( n \) and volume mixing ratio \( x \) for an inert species with a downward flux of \( 6 \times 10^{-7} \) cm\(^2\) sec\(^{-1}\) and a velocity at the lower boundary of \(-5 \times 10^{-3}\) cm sec\(^{-1}\). Results of a numerical model utilizing three grid points per atmospheric scale height and the same boundary conditions are indicated with a dashed line.

decreases monotonically to the tropopause (the lower boundary) (Atreya 1986), the calculated density profile of a long-lived species below the level of its formation increases monotonically with increasing pressure down to the tropopause. This is a consequence of the fact that the downward flux of such a species is constant when atmospheric sphericity is taken into account. On the other hand, 1D models of the terrestrial atmosphere typically adopt profiles of \( K(z) \) of the form shown in Fig. 1a—a rapidly mixed troposphere and a stagnant lower stratosphere. Two solutions have been published for stratospheric profiles of long-lived species with constant downward flux given such a \( K(z) \) profile. Assuming that the species density varies continuously over the whole altitude range, Hunten (1975b) finds that the density profile has a maximum above, not at, the tropopause. More recently, Strobel (1983) adopts the condition that the flux is constant through the tropopause and finds the same solution as in the case of the model with only the stratosphere considered.

In the Appendix we present an analytic solution to the case of the three-segment piecewise continuous eddy diffusion profile that approximates the profile in Fig. 1a in the limit of the middle segment vanishing. Since \( K(z) \) is continuous, the calculated species density profile is also continuous. In Fig. 1b, we show the analytic solution near the tropopause for the abundance of an inert species with a downward flux of \( 6 \times 10^{-7} \) cm\(^2\) sec\(^{-1}\) and \( K(z) \) shown in Fig. 1a. The velocity at the lower boundary has been set to \(-5 \times 10^{-3}\) cm sec\(^{-1}\). Note that the maximum of the density profile occurs \( \sim 1/k \) atmospheric scale heights above the "tropopause," i.e., location of discontinuity in eddy diffusion profile. This confirms the solution presented by Hunten (1975b). Nowhere in the lower stratosphere and upper troposphere does the volume mixing ratio approach a constant value with altitude.

The contrast between the maximum abundance in the lower stratosphere and the tropospheric values reflects the values of several parameters. While changes in the tropospheric eddy diffusion coefficient \( K_t \) result in small changes in the lower stratosphere abundance profile, the tropospheric abundance is inversely proportional to \( K_t \) as discussed in Strobel (1983). Increasing the scale height of the stratospheric eddy diffusion coefficient, by \( \sim 50\% \) for example, increases slightly the maximum abundance in the lower stratosphere and the height of the maximum above the tropopause. Given a constant ratio between the minimum stratospheric eddy diffusion value and \( K_t \), the absolute profile will scale with \( K_t \) as discussed above. Therefore the contrast between the stratosphere maximum concentration and troposphere concentration will remain unchanged. Similarly, the profile scales throughout the whole altitude range with the choice of the magnitude of the downward flux.

An important question is whether complex 1D chemical models of planetary atmospheres adopting discontinuous eddy diffusion profiles of the form shown in Fig. 1a can achieve the correct solutions of the form in Fig. 1b or whether significant numerical diffusion is introduced by the adopted computer algorithm. Superimposed on the analytic profile in Fig. 1b are the results from the Caltech/JPL 1D general planetary atmosphere model (Allen et al. 1981) using the same parameters as the analytic calculation. With finite difference grid intervals 3 to an atmospheric scale height, the numerical model yields values \( \sim 10\% \) less than the analytic model. Increasing the interval density to 30 per atmospheric scale height leads to model results only a few percent less than the analytic values.

ETHANE AND CARBON MONOXIDE ABUNDANCES NEAR THE JOVIAN TROPOPAUSE

Since the Galileo mission will measure stratospheric and tropospheric species abundances, it is of interest to ask what kind of information would be most useful for advancing our understanding of the Jovian atmosphere. We intend to answer this question by carrying out two simplified model studies of ethane (\( \text{C}_2\text{H}_6 \)) and carbon monoxide (CO) in the lower stratosphere and upper troposphere.

Ethane is produced in the mesosphere of Jupiter from the photodissociation of \( \text{CH}_4 \) (Strobel 1969). It is virtually inert throughout most of the stratosphere and tropo-
sphere, and is ultimately converted back to CH$_4$ in the interior of the planet. In a simple model we may assume a mesospheric source $\phi$, a stagnant stratosphere, and a rapidly mixed, diffusive troposphere. There also may be a deep atmosphere source of C$_2$H$_6$ (Prinn and Barshay 1977), such as will be discussed next for CO.

The distribution of CO in the Jovian atmosphere poses a slightly more complicated problem. Proposed extraplanetary sources of CO derived from oxygen atoms include oxygen originating from the erosion of incoming carbonaceous chondritic meteorites (Prather et al. 1978) and from Io and other Galilean satellites (Strobel and Yung, 1979). There is also a source of CO derived from equilibrium chemistry between H$_2$O and CH$_4$ in the interior of the planet (Prinn and Barshay 1977). In the intermediate regions, the stratosphere and troposphere, CO is essentially an inert tracer; CO will not be affected by the clouds in the region 10 to 0.1 bar (Carlson et al. 1987).

We ran a simple model in which we prescribed the downward fluxes of C$_2$H$_6$ and CO in the midstratosphere and their abundances in the deep troposphere. At the upper boundary at 0.5 mbar, a downward flux of $1.2 \times 10^9$ cm$^{-2}$ sec$^{-1}$ was chosen for C$_2$H$_6$ to reproduce the observed volume mixing ratio of $5 \times 10^{-6}$ at $\sim 1$ mbar (Noll et al. 1986). The upper boundary condition for CO, a downward flux of $9 \times 10^9$ cm$^{-2}$ sec$^{-1}$, yields a CO column density above the tropopause of $\sim 4 \times 10^{17}$ cm$^{-2}$, which is consistent with the stratospheric upper limit of Noll et al. (1988) and the detected column density of $\sim 4 \times 10^{17}$ cm$^{-2}$ reported by Beer (1975) and Beer and Taylor (1978), who concluded all the CO was present in the stratosphere. The lower boundary conditions were chosen to conform to the thermochemical calculations of Prinn and Barshay (1977). According to this work, the thermodynamic equilibrium abundance in the upper troposphere is restricted to the abundance of some deeper pressure level at which the chemical lifetime of the species is equal to the time constant for vertical mixing. (At lower pressures and lower temperatures, the chemistry "freezes out.") Therefore, for different tropospheric eddy diffusion coefficients, we adopted the corresponding CO mixing ratios from Fig. 2 of Prinn and Barshay (1977) for the CO lower boundary condition and, in turn, determined the pressure level of the lower boundary, the "freeze out" level, from their Fig. 1. Assuming that the chemical time constants for CO and C$_2$H$_6$ are comparable (and consequently the freeze out levels are similar), the C$_2$H$_6$ mixing ratio at the lower boundary also was obtained from their Fig. 1. There are clearly uncertainties in these adopted boundary conditions which will affect the model results, but not the general conclusions of this paper.

We defined a typical Jovian atmosphere for use in these calculations. A hydrostatic background atmosphere was prepared by utilizing temperatures ($T$) based on Voyager IRIS and Radio Science measurements (W. Maguire, private communication, 1984) and an adiabatic extrapolation to the deep troposphere [$T = 172$ K at 1 mbar, 107 K at 100 mbar, 1266 K at $10^3$ bar; see Figs. 2.5 and 2.6 in Atreya (1986)]. We adopted an eddy diffusion profile with a minimum value at the tropopause (100 mbar) of $2 \times 10^5$ cm$^2$ sec$^{-1}$, varying with increasing altitude as $K(z) \propto M^{-1/2}$ (where $M$ is total atmospheric number density), as suggested by theory and observations (Lindzen 1971, Atreya et al. 1981, Fricke et al. 1982). At the homopause ($M \sim 1.4 \times 10^{15}$ cm$^{-3}$), the resulting eddy diffusion coefficient is $1.4 \times 10^6$ cm$^2$ sec$^{-1}$ as derived from Voyager measurements by Atreya et al. (1981) and McConnell et al. (1981). The eddy diffusion coefficient below the tropopause was greater than or equal to the minimum stratospheric value.

In a series of model runs, we varied the tropospheric eddy diffusion coefficient $K_1$ between $2 \times 10^5$ and $10^8$ cm$^2$ sec$^{-1}$. Figure 2 shows the calculated model profiles for C$_2$H$_6$ and CO for $K_1 = 10^4$ and $10^8$ cm$^2$ sec$^{-1}$. These values reflect the range suggested in earlier studies: $K_1 \sim 10^4$ to $10^5$ cm$^2$ sec$^{-1}$ in the upper troposphere from photochemical models (for example, Strobel 1977) and $K_1 \sim 10^7$ to $10^8$ cm$^2$ sec$^{-1}$ in the deep free convection region (Stone 1976). As seen in these figures, the mixing ratios of downward flowing chemically unreactive species decrease by several orders of magnitude in the lower stratosphere. These calculations predict mixing ratios for C$_2$H$_6$ and CO at 5 bars on the order of $10^{-10}$ to $10^{-8}$.

The species profiles between $10^2$ mbar and $10^3$ bar are drastically different depending upon the choice of $K_1$. In particular, the tropospheric abundance may change by more than two orders of magnitude over this pressure range.
range at the smaller value of $K_1$, but become constant with altitude at the larger value of $K_1$. This is the result of the relative strengths of the upper atmosphere and deep atmosphere sources for each molecule. For large $K_1$, the mixing process constrains the whole troposphere to be fixed to the deep atmosphere value. For small $K_1$, the downward flux of species from the stratosphere pulls the tropospheric solution away from the constant deep atmosphere mixing ratio, the magnitude of this effect being related to the magnitude of the downward flux.

In their observations, obtained with higher spectral resolution than previous Jovian CO observations, Noll et al. (1988) can distinguish between CO located in the stratosphere and in the troposphere. They determined a volume mixing ratio of $1.6 \times 10^{-9}$ for the pressure range 2–9 bar, from which they conclude that the CO source is rapid upward mixing from the deep atmosphere. Figure 3a displays the CO mixing ratios at 5 bar from all of our constant tropospheric eddy diffusion coefficient model runs and shows that there are two scenarios leading to the observed tropospheric mixing ratio. For $K_1 \approx 6 \times 10^3$ cm$^2$ sec$^{-1}$, the upper tropospheric CO is an accumulation of downward flowing photochemically derived stratospheric CO, whereas for $K_1 = 10^8$ cm$^2$ sec$^{-1}$, the result is consistent with the Noll et al. (1988) analysis of their observational data. Interestingly, an analysis of the observed parahydrogen fraction suggests an eddy diffusion coefficient for the upper troposphere (at the Jovian equator) of $3 \times 10^3$ cm$^2$ sec$^{-1}$ (Conrath and Gierasch 1984).

The CO tropospheric profile is, however, sensitive to the downward flux of CO from the stratosphere as noted above. If the flux upper boundary condition for CO was reduced by a factor of 10 to 100, the calculated stratospheric abundance of CO would be reduced proportionately, consistent with the discussion of Larson et al. (1978) contesting the stratospheric origin of the detected CO. The upper tropospheric profile would also decrease accordingly. Then for low values of $K_1$, the tropospheric CO would be significantly below the Noll et al. (1988) observations.

While the CO mixing ratio does not uniquely define a tropospheric eddy diffusion coefficient and species source region, observations of tropospheric C$_2$H$_6$ may be able to distinguish between potential scenarios. As seen in Fig. 2, the C$_2$H$_6$ mixing ratios are very different at 5 bar while the CO mixing ratios are quite similar. This is further illustrated in Fig. 3b where we display the CO/C$_2$H$_6$ abundance ratios at 5 bar from the various constant tropospheric eddy diffusion numerical model runs. The CO/C$_2$H$_6$ ratio differs by a factor of 400 between the two cases for which the CO mixing ratio matches observations.

We have also considered a two-layer troposphere, in which case the eddy diffusion coefficient is a constant $10^4$ cm$^2$ sec$^{-1}$ between 100 mbar and 20 bar and is constant, but a different value, below 20 bar. We carried out a series of model runs in which the deep troposphere eddy diffusion coefficient varied between $10^4$ and $10^8$ cm$^2$ sec$^{-1}$. The case for $10^4$ cm$^2$ sec$^{-1}$ is actually the same as the previous one-layer troposphere model. The case in which the deep troposphere value is $10^8$ cm$^2$ sec$^{-1}$ is a scenario that is consistent with both sets of previous inferences of tropospheric eddy diffusion values, a rapidly mixed deep atmosphere and a less rapidly mixed upper troposphere. The CO and C$_2$H$_6$ mixing ratios calculated for this case are shown in Fig. 4. Figure 3 presents the CO mixing ratio and CO/C$_2$H$_6$ abundance ratio at 5 bar, respectively, from each of the two-layer troposphere model runs. As seen in Fig. 3a (also by comparing Fig. 4 with Fig. 2), the CO abundance in the upper troposphere in the two-layer scenario is elevated over values in the one-layer scenario, suggesting that the downward flux of photochemically derived CO makes a noticeable addition to the abundance mixed up from the deep interior. The Noll et al. (1988) CO observation is again reproduced with a deep atmosphere eddy diffusion coefficient of $10^8$ cm$^2$ sec$^{-1}$ and, therefore, is not uniquely indicative of the eddy diffusion profile of the whole troposphere. Indeed, if the downward stratospheric CO flux is somewhat larger, the CO troposphere observation would be in agreement with a deep troposphere eddy diffusion coefficient less than $10^8$ cm$^2$ sec$^{-1}$ in a two-layer model.
The $\text{C}_2\text{H}_6$ profile in the upper troposphere in the two-layer troposphere model, as shown in Fig. 4, is clearly different from the profiles in the one-layer troposphere models (Fig. 2) that are the limiting cases. This is also revealed in Fig. 3b where the CO/$\text{C}_2\text{H}_6$ ratios from the two-layer calculations are distinctly smaller than the comparable one-layer results. Therefore, all three cases of similar CO abundances at 5 bar can be distinguished by the $\text{C}_2\text{H}_6$ abundance at 5 bar.

**CONCLUSIONS**

A model including both the stratosphere and troposphere of Jupiter may yield abundance profiles distinctly different from profiles generated by models considering only the stratosphere or only the troposphere. The new profiles from the troposphere–stratosphere model have important implications for future observations.

In a one-dimensional model incorporating a stagnant stratosphere layer overlying a rapidly mixed troposphere layer, an analytic model shows conclusively that the concentration profile of an unreactive species flowing downward with a constant flux has a maximum value approximately one scale height above the tropopause. The variation between the lower stratosphere maximum and troposphere concentrations is directly related to the variation between the lower stratosphere and upper troposphere eddy diffusion coefficients. Reflecting the sensitivity of model results to the adopted species profile shape, our calculated downward flux of CO necessary to produce a stratospheric column density $\sim 4-5 \times 10^{17} \text{cm}^{-2}$ is $\sim 4$ times larger than the estimate of Strobel and Yung (1979), who adopted a monotonically increasing CO profile in the lower stratosphere. Our result is about a factor of 3 smaller than the Noll et al. (1988) estimate because of their choice of an eddy diffusion coefficient for the lower stratosphere a factor of 3 larger than in our model, and is within the range reported in Prather et al. (1978) for a range of eddy diffusion coefficients.

The volume mixing ratio profile for a downward flowing unreactive species in the coupled troposphere–stratosphere model shows a rapid decrease in the lower stratosphere. Therefore, a constant mixing ratio in the stratosphere is not a valid assumption for profile retrievals from observational data. Consequently, published estimates of column mole fraction are probably unreliable. For each retrieval concentration or mixing ratio, the altitude (or pressure) level corresponding to the peak of the contribution function (and the width of the contribution function) needs to be reported.

High-altitude photochemical sources for CO and $\text{C}_2\text{H}_6$ can result in a considerable flux of species from the stratosphere to the troposphere. Model calculations considering both the photochemical sources and deep troposphere thermochemical sources reveal situations in which the photochemical sources can be important/dominant contributors to the upper (visible) troposphere abundances. Such is the case for CO, so that the recently reported measurements of the tropospheric abundance (Noll et al. 1988) do not necessarily reveal the tropospheric eddy diffusion coefficient, the deep troposphere CO abundance, and the bulk planet oxygen abundance. However, among the cases considered in which the calculated CO mixing ratios at 5 bar are very similar, the calculated $\text{C}_2\text{H}_6$ mixing ratios at 5 bar have significant variation. Lightning production of disequilibrium species in upper troposphere clouds may also reproduce the CO observation (Podolak and Bar-Nun 1988), and may have its own characteristic CO/$\text{C}_2\text{H}_6$ ratio. The reader should note, however, the current disagreement in the literature about the effectiveness of lightning synthesis [for example, see Lewis and Fegeley (1984)].

This understanding of the consequences of troposphere–stratosphere interactions on species abundances suggests a series of considerations for future observations. With very high spectral resolution, Noll et al. (1988) demonstrated the capability of separating the troposphere and stratosphere components of the CO distribution. This needs to be done for other species, for example, $\text{C}_2\text{H}_6$. Ultrahigh spectral resolution techniques such as that employed by Kostiuk et al. (1987) may be able to detect the presence of the predicted lower stratosphere maximum and/or structure in the species distributions in the upper troposphere that will be quite diagnostic of eddy diffusion coefficients near the tropopause. Since the assumption of constant stratosphere or troposphere mixing ratios is invalid, the model profiles demonstrate the need for less
than one-scale-height spectral resolution from limb-sounding instruments on future spacecraft missions to Jupiter for making measurements of even unreactive chemical species. The next mission to Jupiter, Galileo, carries a neutral mass spectrometer (NMS) as part of the probe experiment component, but only measurements below the tropopause are planned (Hunten et al. 1986). Our calculations suggest that the NMS does not have the sensitivity to measure the detailed vertical distribution of CO and C_2H_6, but by use of enrichment cells may detect these species near 5 and 10 bar. The abundance of CO, the relative CO/C_2H_6 abundance ratio, and the variation of these quantities as a function of pressure level will reveal important information about atmosphere mixing processes and high-altitude and deep troposphere chemistry.

**APPENDIX**

Vertical transport in a one-dimensional model of an inert species is governed by the continuity equations (Hunten 1975a, Strobel 1983)

$$\frac{db}{dz} = 0$$  \hspace{1cm} (A1)

and

$$\phi(z) = -K \left[ \frac{dn}{dz} + \frac{n}{H_k} \right]$$  \hspace{1cm} (A2)

where flux $\phi$ is negative when downward flowing, $z$ is altitude, $n$ is number density, $K$ is the eddy diffusion coefficient, and $H_k$ is the scale height of the background atmosphere. Expression (A2) is for the special case of an isothermal atmosphere well below the homopause, the latter point allowing molecular diffusion to be neglected. For an atmospheric layer characterized by a single value for $H_k$ and in which eddy diffusion is exponentially increasing with altitude,

$$K(z) = K \exp(z/H_k)$$  \hspace{1cm} (A3)

where $H_k$ is the eddy diffusion scale height, the solution of expressions (A1) and (A2) is (Hunten 1975a, Strobel 1983)

$$n(z) = C \exp(-z/H_k) - \frac{\phi_0}{K(z) \left( \frac{1}{H_k} - \frac{1}{H_z} \right)}$$  \hspace{1cm} (A4)

where $C$ is determined by boundary conditions and the constant value for flux $\phi_0$ is regulated by processes outside of the layer. The first term of the right-hand side of (A4) may be referred to as the “hydrostatic” term and the second term is due to the downward flux.

Now let us consider a three-layer atmosphere, all three layers having the same density scale height, where $K$ has the following form:

$$K(z) = K_1$$  \hspace{1cm} $z \leq z_1$

$$= K \exp(-z/H_z)$$  \hspace{1cm} $z_1 < z \leq z_2$

$$= K \exp(+z/H_z)$$  \hspace{1cm} $z_2 < z$  \hspace{1cm} (A5)

The eddy diffusion scale heights for each layer of the atmosphere are $H_{z1}$, $H_{z2}$, and $H_{z3}$, respectively; in this case, $H_{z3} = \infty$. As defined in this manner, $K(z)$ is piecewise continuous, but, in the limit of $H_{z3} \rightarrow 0$, is equivalent to the discontinuous profiles typical of troposphere-stratosphere 1D models, where $K_1 > K_2 = K_3 = \infty$. For example.

Given a velocity $v_0$ through the lower boundary of the lowest layer, the number density $n_0$ at this lower boundary is $\phi_0/v_0$. By requiring $\phi(z)$ to be continuous, the solution for the number density profile in each layer is given by

$$n(z) = A_1 \exp\left(-\frac{z}{H_z}\right) - \frac{\phi_0}{K_1}$$  \hspace{1cm} $z \leq z_1$

$$= A_2 \exp\left(-\frac{z}{H_z}\right) - \frac{\phi_0}{K(z) \left( \frac{1}{H_z} + \frac{1}{H_{z2}} \right)}$$  \hspace{1cm} $z_1 \leq z \leq z_2$  \hspace{1cm} (A6)

$$= A_3 \exp\left(-\frac{z}{H_z}\right) - \frac{\phi_0}{K(z) \left( \frac{1}{H_z} - \frac{1}{H_{z2}} \right)}$$  \hspace{1cm} $z_2 \leq z$

where

$$A_1 = \phi_0 \left( \frac{1}{v_0} + \frac{H_{z1}}{K_1} \right)$$  \hspace{1cm} (A7)

$$A_2 = A_1 - \frac{\phi_0}{K(z_1) \left( H_{z2} + H_z \right)} \exp\left(\frac{z_1}{H_z}\right)$$  \hspace{1cm} (A8)

and

$$A_3 = A_2 - \frac{\phi_0}{K(z_1) \left( \frac{1}{H_{z1}} - \frac{1}{H_{z2}} \right)} \exp\left(\frac{z_2}{H_z}\right)$$  \hspace{1cm} (A9)

**ACKNOWLEDGMENTS**

The assistance provided by R. Achterberg, D. Hunten, D. Strobel, and M. Summers is gratefully acknowledged. The research was supported by NASA Grant NSG-7376 to the California Institute of Technology.

**REFERENCES**


Conrath, B. J., and P. J. Gierasch 1984. Global variation of the


