SEPARATED OSCILLATORY FIELD MEASUREMENTS ON FAST HYDROGEN

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An analysis of the line shape obtained in separated oscillatory field measurements of the hydrogen \( n = 3 \) Lamb shift is presented. The non-ideal interaction geometry is shown to be capable of producing line distortions on the order of 1% of peak quenching in such experiments on unstable levels. Other lineshape details of interest for precision work are explored.

1. Introduction

With a fast atomic beam, it is possible to extend the many advantages of atomic-beam spectroscopy to the study of atomic states whose short lifetimes make them inaccessible at thermal velocities. Spectroscopic lineshapes obtained in such experiments are conceptually simple and are unobscured by velocity averages. In addition, such experiments offer the unique opportunity to observe resonance lines significantly narrower than their "natural width" by means of the atomic beam method of separated oscillatory fields\(^1,2\). With this technique, an interference term involving two spatially separated interaction regions selects atoms living longer than a natural lifetime, thus beating the uncertainty principle for the spectroscopic linewidth. While engaged in a series of experiments aimed at improving knowledge of the hydrogen Lamb shifts, we have carried out a detailed analysis of the lineshapes obtained in this type of experiment. We have paid special attention to the distorting effects of the fringe-field regions which may assume non-negligible proportions in precision experiments with unstable states. Section 2 below presents the apparatus used in this experiment, while section 3 outlines the principles of the method assuming ideal interaction geometries. In section 4 the modifications in lineshape produced by the real experimental geometry are explored.

2. Experimental apparatus

A level diagram for the \( n = 3 \) hydrogen Lamb shift is shown in fig. 1. In this experiment, for simplicity, the line is observed at zero magnetic field where, for a uniformly polarized spectroscopy field, it consists of 4 hyperfine components at 3 different frequencies. The natural linewidth for these transitions is such that they strongly overlap. In what follows, no attempt is made to alter experimentally the populations of the hyperfine states, since for the purpose of investigating lineshape systematics, the natural ratios are preferable. Transitions between S and P states are detected by their different lifetimes (160 and 5 ns) and decay paths (Balmer \( \alpha \), Lyman \( \beta \)).

The basic experimental outline is shown in fig. 2. A beam of 15–20 kV hydrogen atoms with a significant population of the \( n = 3 \) level is produced at left. With a beam of this speed, although the P state will decay almost immediately, the S states can survive to the photo-tube at right where they are monitored by the level of Balmer \( \alpha \) emission. In the rf interaction region, the atoms are subjected to a radio frequency electric field which, when driven near the frequency of the S–P separation, can cause atoms in the S state to make a transition to the P state midway through the

Fig. 1. Level diagram of hydrogen \( n = 3 \) Lamb shift in zero magnetic field. Double arrows indicate allowed transitions for \( Z \) polarized rf field.

Fig. 2. Block diagram of experimental apparatus.

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apparatus, resulting in a decrease in the Balmer $\alpha$ production at right.

In fig. 3 is shown the source of fast hydrogen atoms. Protons are extracted from an rf ion source, focused by an electrostatic einzel lens and passed through a differentially pumped charge exchange cell to produce excited hydrogen atoms. With a few mtorr of nitrogen in the cell, beam neutralization was about 50% and system pressure was near $10^{-6}$ torr. We preferred the charge exchange cell over other means of charge capture since it was convenient, durable, preserved the beam collimation, and by its length averaged out phase coherences in the beam populations.

Fig. 4 shows two examples of rf interaction regions used in this experiment. Both are designed as sections of 50 $\Omega$ transmission line to make possible reliable measurements of the field acting on the atoms over a wide frequency range by means of a broad band power meter at the exit port. The top chamber is of unbalanced design, with one side maintained at ground potential, while the bottom chamber produces a balanced field configuration about the beam line when the two ports are driven out of phase. In practice two identical regions are sequentially encountered by the beam, with the distance between them determining the linewidth of the interference term. The system shown in fig. 5 drives these two regions in such a way that they are both off, both on and in phase, or both on and out of phase in a timing sequence synchronous with the gate pulses to three separate counters which monitor the photo-tube current. The workhorse of the rf system is a
precision 180° hybrid or “magic tee” which delivers two outputs of equal amplitude and of 0° or 180° relative phase depending on which of the input ports is selected. When operating with the balanced rf chambers, this unit is replaced by a “monopulse comparator” which delivers 4 outputs of the proper phase and amplitude. When the rf frequency is varied in the neighborhood of the Lamb shift, the three recorded count rates yield the “0 and π signals”, i.e., the fractional quenching produced when the two regions are in and out of phase respectively. The difference between these constitutes the interference signal. Data from a typical scan across the $n=3$ Lamb shift complex are shown in fig. 6. Statistical errors are smaller than the plotted points and the smooth line is the prediction of the lineshape theory developed below. The separation between the fields has been kept small to facilitate the lineshape analysis.

3. Principles of the method (ideal geometry)

Explanation of this data begins with the semi-classical Schroedinger’s equation for oscillatory coupling between two isolated levels, which is reproduced in eq. (1):

$$i \frac{d}{dt} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} \omega_1 - \frac{1}{2} i \Gamma_1 & 2V \cos(\omega t + \delta) \\ 2V \cos(\omega t + \delta) & \omega_2 - \frac{1}{2} i \Gamma_2 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix};$$

$$V = \frac{e E_0}{2\hbar} \langle \psi_1 | e \cdot r | \psi_2 \rangle;$$

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} e^{+i\omega t} & 0 \\ 0 & e^{-i\omega t} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix};$$

$$i \frac{d}{dt} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \approx \begin{bmatrix} \omega_1 - \frac{1}{2} i \Gamma_1 & Ve^{-i\delta} \\ Ve^{+i\delta} & \omega_2 + \frac{1}{2} i \Gamma_2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix};$$

(1)

The diagonal elements in the Hamiltonian account for free precession and phenomenologically for the radiative decay of both levels. A simple device for approximate solution of this problem is application of the time dependent transformation shown, which may be viewed as transformation to a “rotating frame”. The transformed Hamiltonian describes states separated by $(\omega_0 - \omega)$ and coupled by a static field and an additional anti-resonant field with frequency $2\omega$. Neglect of the anti-resonant term constitutes the rotating field approximation which, with eventual allowance for a small “Block–Siegent shift” in the line...
center\(^4\), is adequate for our purposes\(^4\). A solution of the simple time independent equations takes the form of the evolution matrix shown in eq. (2)

\[
\begin{bmatrix}
a_1(t) \\
a_2(t)
\end{bmatrix} = e^{-\left(t\tau + i\Omega\right)} \times \\
\begin{bmatrix}
\cos(p + \frac{1}{2}at) & -ie^{-i\beta} \sin(\frac{1}{2}at) \\
-ie^{i\beta} \sin(\frac{1}{2}at) & \cos(p - \frac{1}{2}at)
\end{bmatrix}
\begin{bmatrix}
a_1(0) \\
a_2(0)
\end{bmatrix},
\]

where

\[
a = \sqrt{\left(\Omega - \frac{1}{2}iQ\right)^2 + (2V)^2},
\]

\[
p = -i \ln \left[ \frac{a - (\Omega - \frac{1}{2}iQ)}{2V} \right],
\]

\[
\bar{\Omega} = \omega_0 - \omega, \quad \bar{\beta} = \frac{1}{2}(\omega_1 + \omega_2)
\]

\[
\Omega = \Gamma_1 - \Gamma_2, \quad \bar{\Gamma} = \frac{1}{2}(\Gamma_1 + \Gamma_2).
\]

(2)

For stable levels, this solution reduces to that of Ramsey\(^1\). For weak coupling and interaction times longer than the shortest lifetime involved, the resonance terms that result are Lorentzian-like with a half width approaching \(Q\). For other conditions, such as those encountered in this experiment, the line-shapes are decidedly non-Lorentzian.

The principle of the interference scheme can be understood by considering two such interaction regions separated by a region of free space as shown in fig. 7. The experimentally measured quantity is the square of the amplitude for an atom which enters as an S-state to leave in the same condition. This result can be realized by two paths which interfere to produce the interesting effect. Each of the path amplitudes is written as the product of its component amplitudes, given explicitly in eq. (2). The whole process is most simply viewed in the rotating frame where it is clear that the relative precession between the two paths takes place at \((\omega_0 - \omega)\). Another important feature is the manner in which the relative phase of the fields enters, providing the means of separating the interference term. Thus the total transition amplitude contains the cosine term shown whose width is decreased by increasing \(T\), and which can be isolated by switching the relative phase of the regions between 0 and \(\pi\). \(f_\perp\) is an odd function of \((\omega_0 - \omega)\) that increases the effective interference separation to roughly the separation between the midpoints of the fields. Fig. 8 gives the theoretical line-shapes for such an ideal experiment using modestly separated fields.

![Fig. 8. Theoretical profiles of 0, \(\pi\), and interference signals for a single hyperfine component, assuming ideal rf fields.](image)

Application of this two-level line-shape to the hyperfine complex under consideration is made possible by the decoupling of the problem into 4 two level pairs under uniform polarization. For the fit shown in fig. 6, the four lines have been superposed with equal amplitude with only the Lamb shift frequency and the overall normalization being adjusted. The agreement is on the order of 1% r.m.s. of peak quenching. To achieve even this degree of quantitative agreement, especially with the \(\pi\) signal, considerable care must be exercised in the choice of parameters \(V\), \(t\) and \(T\) in the

![Fig. 7. Origin of the interference term.](image)
uniform field model. In our case, these parameters were not chosen to obtain best fit at the risk of obscuring other effects, but as a corollary to a study of the effects of the non-ideal geometry. In the process, we were able to make quantitative estimates of several potentially serious sources of line distortion.

4. Modifications arising from the real geometry

The real field distribution in the rf regions differs from the ideal since it is uniform in neither magnitude nor polarization. For this study we have approximated it with a sequence of smaller regions of uniform magnitude and polarization. The 8 × 8 evolution matrix for any polarization is obtained from the block form matrix of Z polarization by rotation. As illustrated in fig. 9, the number of paths available in a typical observed amplitude has been enlarged to include paths forbidden in a field of uniform polarization. These secondary paths contribute to the overall amplitude primarily through their interference with the S₀ path. As can be verified from symmetry considerations, all significant secondary paths change phase relative to the S₀ path between the 0 and π field configurations, making their presence especially significant in a line narrowing interference experiment. Among the secondary paths contributing appreciably to the observed amplitude are those through the P' and S' states illustrated for one case in fig. 9. On axis in the unbalanced rf geometry, P' paths contribute 2–3% of peak quenching and S' paths 0.2–1.0% as compared to 23% of peak quenching from the primary interference term. These shapes are in general geometry-dependent and some are asymmetric. Of course all the secondary path amplitudes vanish on the axis of the balanced rf region where the field really is uniformly polarized, but they grow to comparable magnitude less than 1 cm off axis. Thus careful alignment and moderately close collimation in a balanced rf geometry will be necessary to control these effects to high precision.

Even in the lineshape obtained from the two primary paths the non-idealities can produce significant effects. Specifically we have assumed in section 3 that entry into the field occurs impulsively. In the opposite so-called “adiabatic” limit no interference term can be observed since no coherent superposition of the two eigenstates is ever created. The criterion for impulsive change is well known for static fields; the time scale of the change must be small relative to the period of the energy separation between the states. This condition is directly applicable to rf coupling also as long as the

![Fig. 10. Simulated single component lineshapes using actual rf geometry. For clarity, only half of the symmetric 0 and π signals are shown. The smooth lines are least square fits to a uniform field lineshape where V, t, T and the size of the interference contribution were free parameters.](image)

Fig. 9. Additional interference paths produced by non-ideal geometries.

VII. HIGH-Z SYSTEMS OF ONE AND TWO ELECTRONS; LAMB SHIFT
“energy separation” is taken in the rotating frame where the coupling field is static. The result is that if \( \Delta t \) is a time characteristic of changes in the rf geometry, portions of the line which are off resonance by more than \( 1/\Delta t \) can be expected to show deviations from the impulsive lineshape. This problem is not encountered with stable levels where the resonance linewidth is determined by the time spent in the field, since a reasonable aspect ratio for the plates insures that for all frequencies at which data is taken, the impulsive criterion is satisfied.

We have studied the lineshape due to the two primary paths by approximating it with a lineshape in the uniform field model class, using \( V, t, T \) and the size of the interference part as free parameters. The results for 0, \( \pi \) and interference signals are shown in fig. 10. In the adiabatic region, the structure in the quenching curves vanishes and the interference term decays to zero as expected. For each geometry, the values of the uniform field parameters obtained in this manner were used to obtain working lineshapes for the data fits. Unfortunately, since the residual error in these fits was \( \approx 1\% \) r.m.s. of peak quenching, such uniform field lineshapes are not suitable for high precision work, and a lineshape taking into account the non-impulsive entry conditions off resonance will have to be used.

Another effect observed in the simulated lineshapes obtained from the two primary paths was a slight \( (\leq 0.1\%) \) geometry dependent asymmetry due to the varying field polarizations. This effect is present in both quenching and interference curves and will receive careful study.

5. Summary

We have been able to predict the detailed shape of the separated oscillatory field related lineshapes for the \( n = 3 \) hydrogen Lamb shift complex in zero field to a precision of about 1%. To achieve higher precision, non-impulsive entry conditions must be included in the lineshape theory, and secondary path amplitudes must be carefully controlled.

References

3) F. Bloch and A. Siegert, Phys. Rev. 57 (1940) 522.

Discussion

LEVENTHAL: The real goal is to locate the center of those lines, and distortions do not bother you really in locating the center if they are coming symmetrically, right?

LUNDEEN: Right.

LEVENTHAL: That is what I was going to ask. I could not figure out whether the distortions you talked about came in symmetrically or not.

LUNDEEN: Sorry I did not make that clear. It is in the paper, but I just did not have time to say everything. Some of these distortions are asymmetric. If you remember that slide (fig. 11), some of those additional paths go through states that are at different energies and so obviously those paths are going to have different frequency dependence and some of them will be asymmetric.

LEVENTHAL: Oh, so it shifts then?

LUNDEEN: It turns out that in the experiment that is already been done, all of the additional paths that are significant are symmetric and so do not affect the result. In general, however, they certainly have to be considered in such high precision work.

STONER: Could you tell me again what your beam energy was and whether you know the spread in energy?

LUNDEEN: Well, our beam energy is 15 to 20 kV, and no we do not know the energy spread. But it is small, because we use a gas target, and the effect of the beam energy is smaller than these other effects, for example, the entry conditions. What we expect is that they will start to affect the shape of the curve far off resonance.

STONER: Now you extract ions from an rf discharge. You have a repeller probe in there at some high positive voltage to drive them out. Do you know what that voltage is?

LUNDEEN: That voltage is approximately 2 to 3 kV below the acceleration voltage.

STONER: Is it possible that you have a spread as large as a kilovolt?

LUNDEEN: I do not think so. I think studies of this have shown that the energy spreads are on the order of a few hundred electron volts.